SHOW 108

Soaring Sequences: Thinking About Large Numbers

PROGRAM SYNOPSIS

Segment 1 *(4:23)*
LATE AFTERNOON WITH DAVID NUMBERMAN

Talk show host David Numberman interviews Gloria Johnson, a billionaire ex-waitress. She made her billion in just one month by getting her boss to agree to pay her one dollar the first day, then to double her earnings every day thereafter for the rest of the month.

Segment 2 *(2:38)*
INFINITY— THERE IS NO END

In this music video, the Jets contrast very large numbers and infinity. They give examples of collections that are impractical or impossible to count, but not infinite, such as grains of sand on a beach. The Jets perform against a moving backdrop of number sequences. Each sequence increases by the repeated addition or multiplication of a number.

Segment 3 (3:05)
FAX HEADFULL: WORLD POPULATION

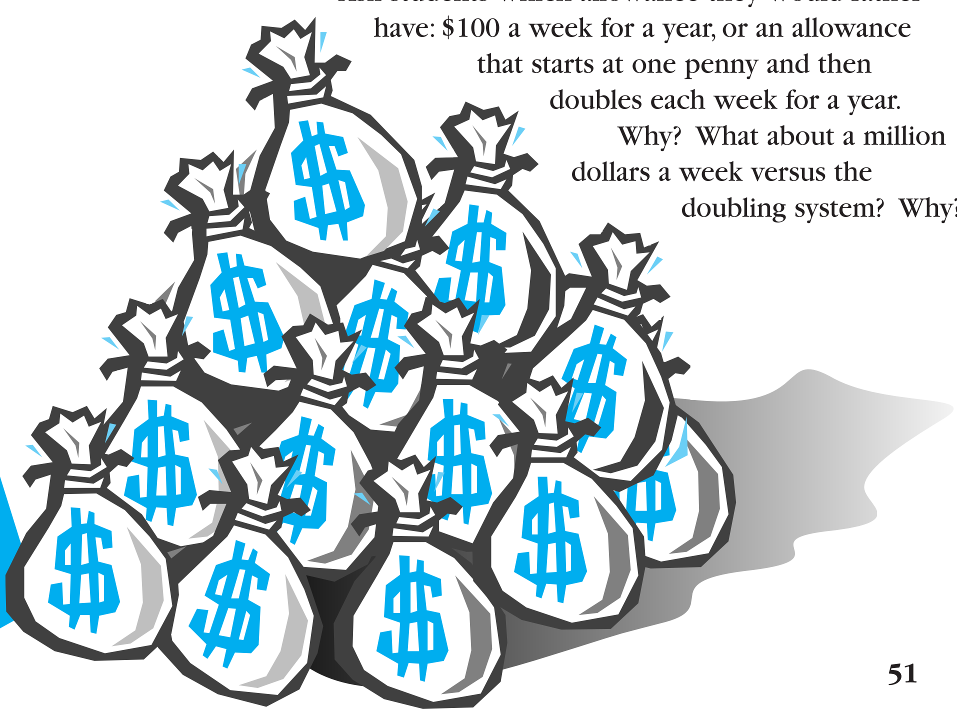
Electronic talking head Fax Headfull talks about a really large number sequence: world population and projections about its growth. Fax illustrates what it would look like if the population doubled every 40 years by transforming himself into the columns of a bar graph representing world population growth.

INTRODUCTION

Large numbers are part of our lives. Understanding what they mean and how they can be generated is important to students' development of number sense. Simple sequential processes can produce large numbers from small ones very quickly. Thus, the ideas of this module span two areas of the NCTM's *Curriculum and Evaluation Standards for School Mathematics* "Number and Number Relationships" (p. 87) and "Patterns and Functions" (p. 98).

BEFORE VIEWING

Ask students which allowance they would rather have: $100 a week for a year, or an allowance that starts at one penny and then doubles each week for a year. Why? What about a million dollars a week versus the doubling system? Why?

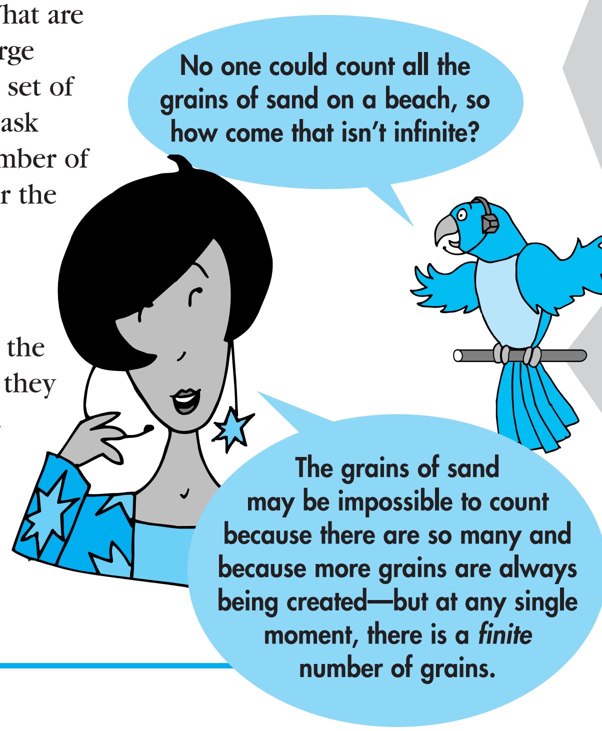


DURING VIEWING

BILLIONAIRE WAITRESS

■ **STOP THE TAPE** at the end of the segment and ask if any students have changed their minds about which allowance they preferred. Why? (The doubling scheme is a much better deal. $100 a week for a year gives $5,200, while doubling, starting with a penny, would give more than a billion dollars after only 30 weeks—and a lot more than that after a year. Even the $1,000,000 a week option yields only $52,000,000 for the year.)

Next, ask students if they think one billion is a very large number. What are other numbers that are even larger? What are some sets that have large numbers of objects in them? (The set of all people in the world, the set of all grains of sand on a beach, stars in the universe, and so on.) Then ask students for examples of infinite sets—sets containing an infinite number of things. Make a list of their ideas on the board. Tell them to watch for the number sequences that roll by during the music video.



INFINITY—THERE IS NO END

■ **STOP THE TAPE** at the end of the song to ask students to review the list of sets that they proposed as infinite sets. See if there's anything they would like to add or remove from it. Encourage them to discuss and defend their ideas. (Things such as the grains of sand on a beach or stars in the sky are not easily counted, but they are not infinite. By contrast, the set of whole numbers, the set of even numbers, the set of fractions between 0 and 1, and so on, are all infinite sets.)

activity

EXPLORING A NUMBER SEQUENCE: A POPULAR LITTLE TOWN

In this activity, students will make a bar graph to use as a tool for predicting the growth of a sequence.

MATERIALS

- copies of reproducible page 56
- calculators

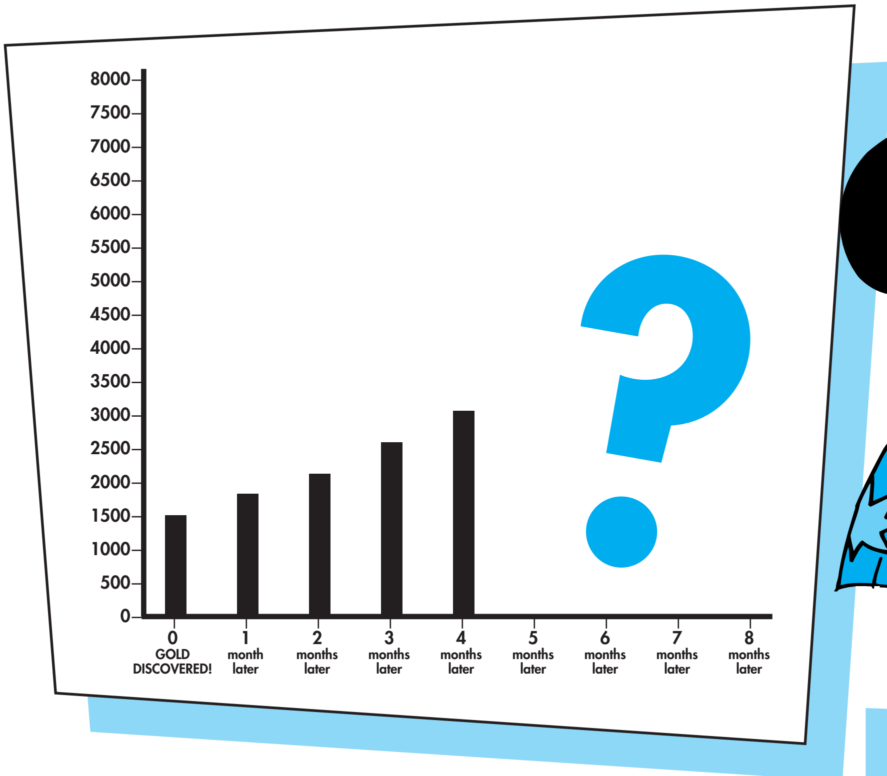
1. Ask the class to imagine Dead Pan, a gold rush town, somewhere in California in about 1850. Dead Pan had a population of 1500 on the day gold was discovered nearby. Word spread fast, and eager prospectors began to move in immediately. Copy the information in this chart onto the chalkboard or an overhead transparency.

   Ask students what they think would be a reasonable prediction for the future growth of Dead Pan.

DEAD PAN, CALIFORNIA

	Population
Gold discovered:	1500
1 month later:	1800
2 months later:	2150
3 months later:	2600
4 months later:	3100

2. Remind students that in the video, Fax Headfull made a bar graph to show what would happen to world population if it continued to grow at its current rate. Either individually or in small groups, students can use the reproducible page to draw a graph of Dead Pan's population data.



One reason for Dead Pan to try to make population projections is so there'll be enough roads, food, water, police, and so on for all the people who'll be living there.

3. Discuss the following: Assuming that the town's population continued to grow as it did during the first four months, how can we use our bar graph to predict the population after another four months? (Predictions will vary according to the ways that students analyze the data. There are many logical ways to do this. Encourage students to look for patterns to explain their reasoning.)

Some possible approaches:

- Since the population about doubled in four months, some students might predict that it would double again in the next four months, to about 6000.
- Some students might think of fitting a straight line along the top of the bars, and extending that line over four more months. Or, having observed the increase of about 1600 in four months, they would predict the same increase for the next four months. Both of these approaches lead to a prediction of about 4700.
- Since the bars in Fax's graph doubled in size at each step, some students may look for a common multiplier for this graph. A calculator will help. They will find that 1.2 works very well: 1.2 x1500 = 1800; 1.2 x 1800 = 2160; 1.2 x 2160 = 2600 (about); 1.2 x 2600 = 3120. This method leads to a prediction of about 6400 after another four months.
- There are other patterns that students might find in these numbers.

Point out that while the different approaches yield different results, a good argument can be made for the validity of each. There's no way to tell for certain what's going to happen to the town's population. In fact, if the gold runs out, the population might drop back to 1500, or even to zero, in a short time.

keep thinking

PLAYING CATCH-UP

Several number sequences go by in the background of the **INFINITY** music video. List them on the board for students. Discuss whether the sets of numbers in those sequences are examples of infinite sets. (They are. Each list will never end because you can always add or multiply one more time.)

Explore these number sequences with the class to help them develop a sense of large numbers and patterns of sequential growth. Calculators will be very useful in this exploration.

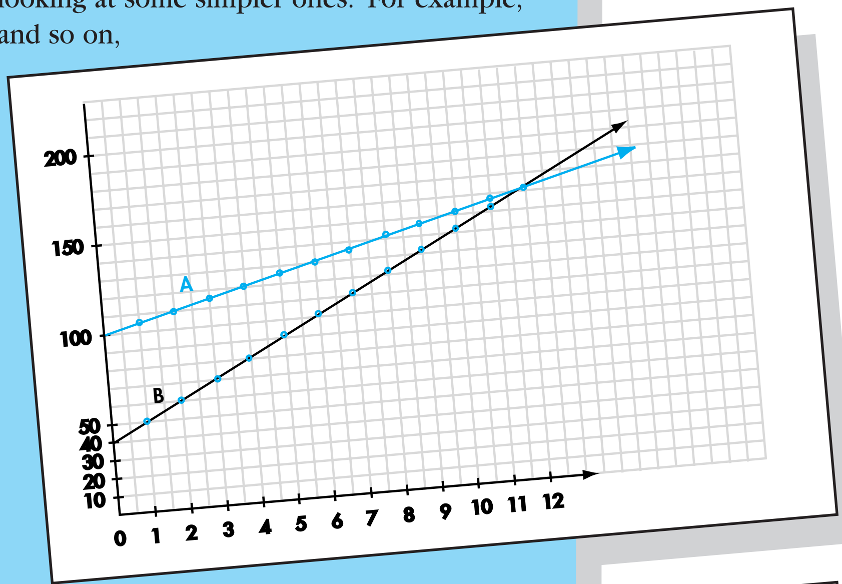
A	B	C	D
8659 + 9	47,403 + 7	362,144 + 3	1 x 2
8668 + 9	47,410 + 7	362,147 + 3	2 x 2
8677 + 9	47,417 + 7	362,150 + 3	4 x 2
8686 + 9	47,424 + 7	362,153 + 3	8 x 2
8695 + 9	47,431 + 7	362,156 + 3	16 x 2

- Begin by comparing the first two sequences. Suppose you continued to list the numbers, side by side. Does A ever overtake B? In other words, will the number in the A column ever be greater than the number in the B column? (Even though A begins with a smaller number than does B, it increases by a slightly larger amount in each step. In fact, A gains 9-7, or 2, on B with every step. No matter how far behind A is, eventually it will catch up.)
- How many steps will it take for A to overtake B? (The sequences differed by 38,744 at the start—that's 47,403-8,659. That initial difference is reduced by 2 on each step. So it will take 38,744 ÷2, or 19,372, steps for A to tie B, and one more step [19,373] for A to go beyond B.)
- Does A ever overtake C? (Yes. A gains by 6 on each step. The initial difference of 353,485 will be overcome in 58,915 steps. In fact, on the 58,916th step sequence A will be ahead of C.)
- Now compare A, B, and C, respectively, with D and ask the same questions. Does D ever overtake A (or B or C)? (Yes. It overtakes each of them.)
- How many steps does D need to overtake A, B, or C? (14, 16, and 19, respectively. The fact that D overtakes the others in so few steps, despite starting so far behind, illustrates once again the power of the doubling sequence.)

FOR THE PORTFOLIO

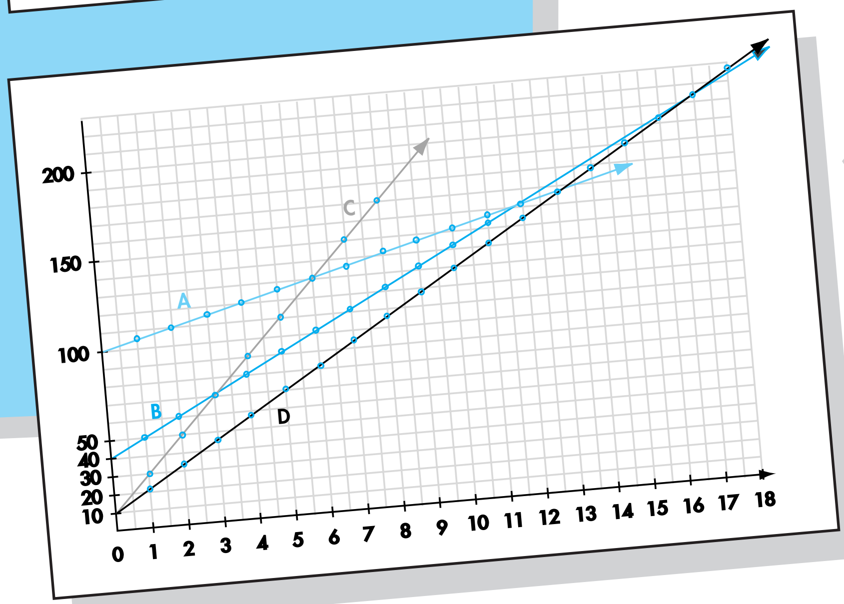
Students may explore, illustrate and write up the following investigation for inclusion in their math portfolios.

You can explore how sequences grow by looking at some simpler ones. For example, suppose sequence A is 100, 105, 110, 115, and so on, starting at 100 and adding 5 each step. Sequence B is 40, 50, 60, 70, and so on, starting at 40 and adding 10 each time. Will B catch up to A? (Yes, because even though it starts behind, it gains 5 each time. They tie at 160, the 12th step.) This can be sketched on a graph, like this:



Can you create a sequence (call it C) that starts at 10, adds the same number at each step, and catches up **first** with B and **then** with A? How about a sequence (call it D) that starts at 10, adds the same number at each step, but catches up **first** with A and **then** with B? (If the number you add is larger than 12.5, then the sequence will catch up with B first, and then A. If the number you add is between 10 and 12.5, the sequence will catch up with A and then B. If the number you add is 12.5 exactly, then there's a three-way tie on the 12th step: All three sequences will be at 160.)

What would C and D look like on the graph?



CURRICULUM CONNECTIONS

This module encourages finding and communicating patterns of various kinds. The numerical patterns hidden in Gloria's charts, the patterns in the sequences of **INFINITY—THERE IS NO END**, and any of the patterns that may underlie the growth of Dead Pan are examples of regularity that students should be encouraged to seek out and explain.

Math Talk	CONNECTIONS	
PROBABILITY & STATISTICS	**The Data Game: Using Graphs**	Bar graphs, line graphs, circle graphs
	Don't Jump to Conclusions: Interpreting Statistics	Using graphical information to make inferences

NAME

SOARING SEQUENCES: THINKING ABOUT LARGE NUMBERS

The chart shows how the population of Dead Pan grew after gold was discovered. Draw a bar graph that shows the town's population when gold was found, and for the next four months. Then continue your graph to make a prediction of what the population will be for months 5, 6, 7, and 8.

DEAD PAN, CALIFORNIA

—Population—

Gold discovered:	1500
1 month later:	1800
2 months later:	2150
3 months later:	2600
4 months later:	3100